Model-Based Sensorless Controller of a Permanent Magnet Synchronous Motor

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Abstract. In this work, it is presented the design and implementation of a Luenberger observer for a permanent magnet synchronous motor (PMSM). A model-based technique is used to control a nonlinear system under a Field Oriented Control scheme (FOC). Moreover, simulation and implementation both, with encoder and sensorless are realized to evaluate the performance of the control. The implementation is achieved using the MCK28335 Kit platform under a experimental environment. In addition, the error criteria IAE, ISE, ITAE and ITSE are explored to evaluate the performance of both control schemes. As a result, a better performance is achieved with sensorless scheme compared the FOC including sensor.

Keywords: model-based controller, permanent magnet synchronous motor, sensorless control.

1 Introduction

Permanent Magnet Synchronous Motor (PMSM) are widely used in many applications (e.g. printers, tape drives, hard drives, process control systems, CNC machine tools, industrial robots, aerospace, electrical vehicles, submarines), due to their efficiency and dynamic performance [7][13][17]. PMSMs have superior features such as compact size, high efficiency, high power density, wide speed ratio, high torque and absence of rotor losses [9][16]. Furthermore, advances magnetic materials having even higher power lead to wider applications of PMSM [17]. In recent years, advancements in magnetic materials and control theories have made PMSMs drives to play an important role in motion-control applications [14].

When the motor mechanical variable (rotor position or speed) are available from measurements, high-precision and robust control of a PMSM can be achieved in rotor position or speed tracking applications for PMSM. A Mechanical and optical sensors are typically used to measure the rotor position or speed for vector control of a PMSM. Nevertheless, these sensors increase hardware complexity, cost and size of the drive systems [6][10]. In addition, the reliability of the drive system is reduced, a regular maintenance is required [1][8]. The disadvantages of mechanical or optical sensors can be removed if the speed can be estimated from the terminal vectors.

Estimation of rotor position or speed without measurement of mechanical variables is a challenging problem for a PMSM. In this framework, a lot of attention has been paid by electric drives community to the "sensorless" control problem of PMSMs in which only stator current and voltage measurements are available for feedback [1][6].

There are two main categories of sensorless control: model-based estimation techniques (e.g. Luenberger and Kalman-Filter observer, full and reduced order closed-loop observer, sliding mode control, model reference adaptive system) and saliency based techniques [1][3][8][19].

Model based techniques can adopt a state observer to retrieve the rotor position information, by extracting the speed information using current quantities obtained from PMSM terminal. It is especially useful for full-state feedback control develop on the state-space theory [1][19].

The sensorless approach has several advantages: 1) only electrical connections to the machine are the main phase windings, so installation cost are minimized; 2) position-sensing function can be located with the others control electronics: it does not need to be sited adjacent to the machine, and therefore does not inhibit the operating temperature range; 3) absence of connecting leads prevents corruptions of position data by electromagnetic interference; and 4) cost of a position encoding device is avoided.

The goal of this paper is to present results for a model-based controller, in order to eliminate the speed sensor on the control law on MCK28335 Kit. The remainder of this paper is organized as follows. In Section 2, presents necessary background about observer and model-based control theory. In Section 3, the dynamical equations, variable and parameters definitions, are described. Also, The design of the proposed controller and the stability analysis are presented. Performance results of simulation and emulation on MCK28335 Kit of proposed controller, are discussed in Section 5. Finally conclusions are given in the last section.

2 Preliminaries

2.1 Luenberger observer

An observer comprises a real-time simulation of the system or plant, driven by the same input as the plant, and by a correction term derived from the difference between the real output of the plant and the estimated one derived from the observer.

Consider a continuous-time linear system expressed by the form:

$$\dot{x} = Ax + Bu; \tag{1}$$

$$y = Cx; \tag{2}$$

where $x \in \mathbb{R}^n$; $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ represents the state, control and output vector; A; B and C are matrices of corresponding dimensions.

Denoting the state vector of the observer by $\hat{x} \in \mathbb{R}^n$, the following state space representation defines the observer:

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} = A\hat{x} + B(u + LC(x - \hat{x})); \tag{3}$$

The error vector is defined as $\tilde{x} = x - \hat{x}$. The error dynamics is described by the following equation:

$$\dot{\tilde{x}} = (A - BLC)\tilde{x}; \quad (4)$$

In order to guarantee convergence of the estimated states, is enough to choose a gain matrix L such that error dynamics converge asymptotically (i.e., when $A - BLC$ is a Hurwitz matrix).

2.2 Model-Based Controller

The model-based control techniques have been largely extended and gained prominence during the past decades, major steps are expected in the future, especially for the non-linear case [2].

Fig. 1 shows the model-based controller, with a full order closed-loop observer's state estimate being fed back through the state feedback gain K.

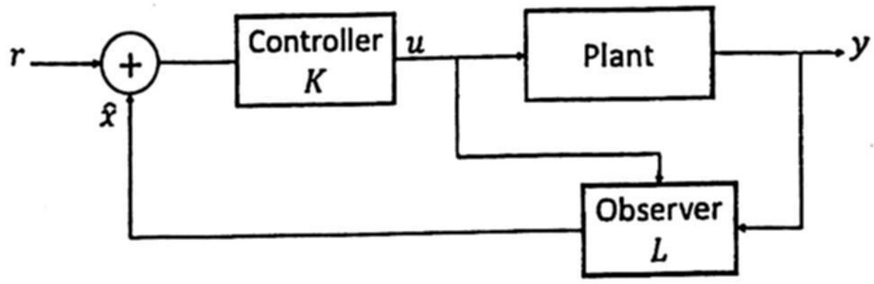

Fig. 1. Closed-loop system using the model-based output feedback controller.

Here, dimension of the plant and controller are the same. Then, the number of state variables of the closed-loop system is twice double that of the open-loop plant.

The plant is given by (1), substituting $u = K\hat{x} + r$:

$$\dot{x} = Ax + BK\hat{x} + Br. \quad (5)$$

To eliminate $\hat{x}$ from (5), this equation is only in terms of the state variable x and $\tilde{x}$, substituting $\hat{x} = x - \tilde{x}$, producing:

$$\dot{x} = (A + BK)x - BK\tilde{x} + Br; \quad (6)$$

Coupling this with (4), we get the composite system's state description:

$$\frac{d}{dt}\begin{bmatrix} x \\ \tilde{x} \end{bmatrix} = \begin{bmatrix} A + BK & -BK \\ 0 & A - BLC \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r. \quad (7)$$

Theorem 1 **(Lyapunov's linearization method [15]).** *If the linearized system is strictly stable (i.e, if all eigenvalues of A are strictly in the left-half complex plane), then the equilibrium point is asymptotically stable (for the actual nonlinear system).*

Since the composite system matrix is a upper triangular block, therefore the closed-loop eigenvaules are given by $\lambda(A + BK) \cup \lambda(A - BLC)$, where, $\lambda(A)$ represents the set of eigenvalues of A. This fact indicates that the plant stabilization and observer design can be approached separately.

3 Model-Based Controller for PMSM

3.1 PMSM model

The dynamical model of a brushless PMSM, according to the reference model based on the Principle of Clarke and Park transformations, is given as follows [18],[20]:

$$L_d\frac{di_d}{dt} = L_q i_q \omega - R i_d + u_d; \tag{8}$$

$$L_q\frac{di_q}{dt} = -(L_d i_d + \psi)\omega - R i_q + u_q; \tag{9}$$

$$J\frac{d\omega}{dt} = m - m_r - F\omega; \tag{10}$$

$$m = \frac{3}{2}\rho(L_d - L_q)i_d i_q + \frac{3}{2}\rho\psi i_q; \tag{11}$$

where i_d and i_q are the stator currents on $d-q$ axis, L_d and L_q are the winding inductances on $d-q$ axis, U_d and U_q are the $d-q$ stator voltages, R is the stator winding resistance, ω is the rotor speed, ψ is the permanent magnet flux, J is the rotor moment of inertia, F is the viscous friction coefficient, and ρ is the number of pole pairs. According with the model represented by (8), it is clearly seen the nonlinearity of a PMSM.

Considering that the motor has a perfect electric symmetry, $L_d = L_q$, and renaming the phase currents as $x_1 = i_d$, $x_2 = i_q$, $x_3 = \omega$ and $m_r = 0$, the system represented by (8)-(11) can be expressed as:

$$\begin{aligned}\dot{x}_1 &= x_2 x_3 - \frac{R}{L}x_1 + \frac{u_d}{L};\\ \dot{x}_2 &= -(x_1 + \frac{\psi}{L})x_3 - \frac{R}{L}x_2 + \frac{u_q}{L};\\ \dot{x}_3 &= \frac{3\rho\psi}{2J}x_2 - \frac{F}{J}x_3;\end{aligned} \tag{12}$$

where, assuming that $u_d = u_q = 0$, and the motor is running freely, the equilibrium point of the system is $x^* = [x_1^*, x_2^*, x_3^*]^T = 0 \in \mathbb{R}^3$. Where x^* denotes the equilibrium point of the system expressed by (12).

3.2 Analysis and design of Luenberger observer

The aim of this paper is to design an observer-based speed controller, in order to eliminate the speed sensor in the control law of MCK28335 Kit. The Field Oriented Control (FOC) scheme for the control system can be seen in Fig. 2, where is shown that this scheme includes an encoder.

Now, in Fig. 3 it is shown the proposed FOC scheme for the model-based control, here the speed encoder is eliminated and substituted by an observer. The structure of the observer is given by (3). Under this scheme, the speed controller is fed with the

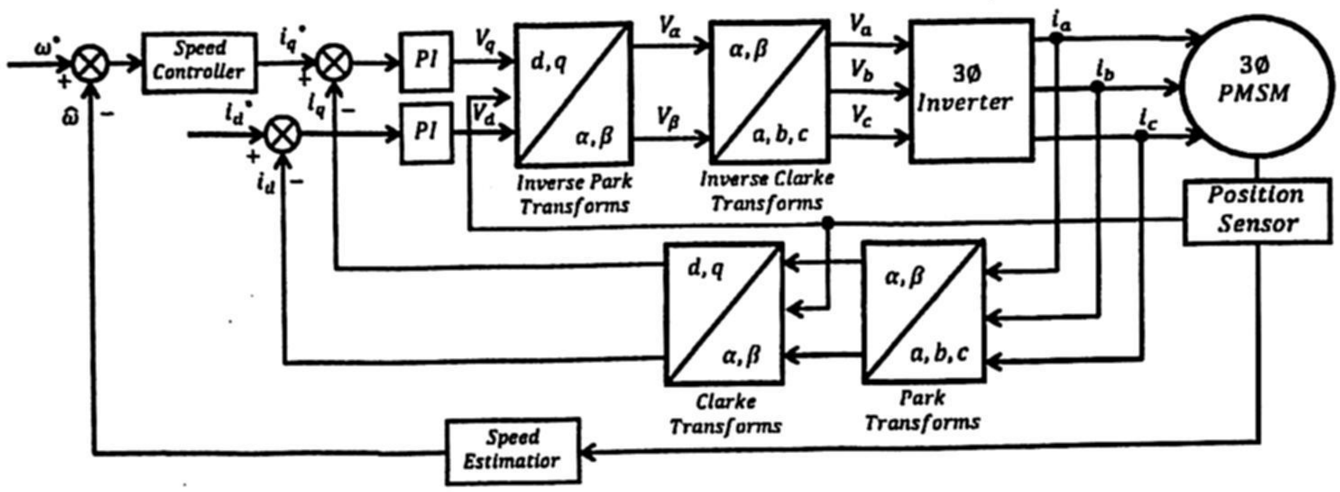

Fig. 2. Field Oriented Control Scheme for PI controller with speed encoder.

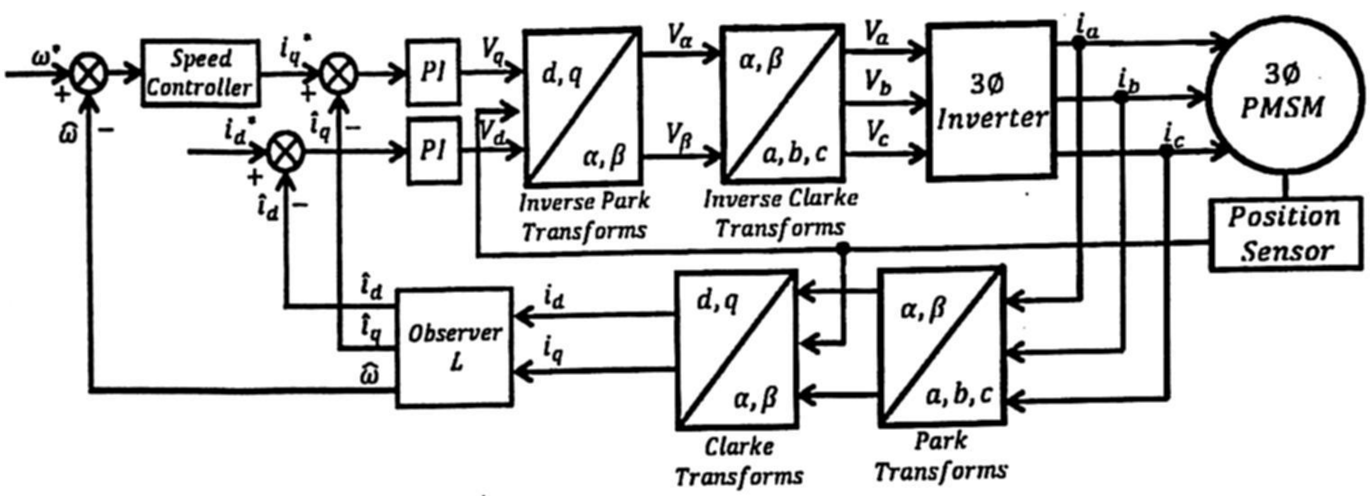

Fig. 3. Field Oriented Control Scheme for the model-based proposed controller.

estimated speed from observer, and the PIs in the inner control loop that control the direct and quadrature axes ($d - q$) are estimated for i_d and i_q.

The FOC scheme of the proposed controller has a similar structure than closed-loop system showed in Fig. 1. The design of the proposed observer is based on the linearization of the dynamic model (12). Where x is the state vector defined as $x = [x_1, x_2, x_3]^T$ and u is the input vector given by $u = [u_d, u_q]^T$. Matrices A and B are defined as follows:

$$A = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & -\frac{R}{L} & -\frac{\varphi}{L} \\ 0 & \frac{3p\varphi}{J} & \frac{F}{J} \end{bmatrix}; \quad B = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix}. \tag{13}$$

The output vector y consist in the measurement of the currents x_1 and x_2 denoted by $y = [x_1, x_2]^T$. Here, the error dynamics matrix $A - BLC$ is given by:

$$A - BLC = \begin{bmatrix} -\left(\frac{R}{L} + \frac{l_1}{L}\right) & -\frac{l_3}{L} & 0 \\ -\frac{l_2}{L} & -\left(\frac{R}{L} + \frac{l_4}{L}\right) & -\frac{\varphi}{L} \\ 0 & \frac{3p\varphi}{J} & -\frac{F}{J} \end{bmatrix}; \tag{14}$$

where according with Theorem 1, in order to ensure the local asymptotic stability, the constant gains are given by:

$$l_1 > -R; \tag{15}$$

$$l_2 = l_3 = 0; \tag{16}$$

$$l_4 > -\frac{3p\varphi^2}{2F}; \tag{17}$$

therefore, with the constant gains l_1 and l_4 can be ensured local asymptotic stability of the observer. The dynamic model of the observer is given by:

$$\frac{d}{dt}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L}\hat{x}_1 + \frac{1}{L}(u_d + l_1(\hat{x}_1 - x_1)) \\ -\frac{R}{L}\hat{x}_2 + \frac{\varphi}{L}\hat{x}_3 + \frac{1}{L}(u_q + l_4(\hat{x}_2 - x_2)) \\ \frac{3p\varphi}{J}\hat{x}_2 - \frac{F}{J}\hat{x}_3 + \frac{1}{L} \end{bmatrix} \tag{18}$$

The stability conditions for the closed-loop system is not addressed due the platform already has implemented a PI controller. According with aforementioned, it is assumed that the PI controller has a good performance and only it is analyzed the observer stability.

4 Simulation and results

The performance of the proposed controller was validated using MATLAB-Simulink software test given by Technosoft, the real-time simulation platform consisting of:

- One three-phase Permanent Magnet Synchronous Motor. Model MBE.300.E500 with parameters shown in Table 1. Equipped with Hall sensors and 500-line encoder for direct experimentation.
- One development system model Technosoft MSK28335 with a DSP motion controller model TMS320F28335.
- One three-phase Voltage Source Inverter (VSI) model PM50 power module board with rated voltage 36 V, rated current 2.1 A, rated power 75 W, DC link voltage in a range of 1236 V, working at PWM frequencies up to 20 kHz.

Numerical simulation of the error dynamics of the proposed observer are shown in Fig. 4. The initial condition of PMSM is $[x_1(0), x_2(0), x_3(0)]^T = 0 \in \mathbb{R}^3$ and the initial condition of (18) is $[\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0)]^T = 1 \in \mathbb{R}^3$.

Fig. 4 shows convergence of the error dynamics of the observer.

In addition, numerical simulations are realized in order to make a comparison between the performance of FOC scheme with encoder and proposed sensorless FOC

Table 1. Parameters of the PMSM.

Coil dependent parameters		
Name	Value	Units
Phase-phase resistance	8.61	Ω
Phase-phase inductance	0.713	mH
Back-EMF constant	3.86	$V/1000rpm$
Torque constant	36.8	mNm/A
Pole pairs	1	
Dynamic parameters		
Rated voltage	36	V
Max. Voltage	58	V
No-load current	73.2	mA
No-load speed	9170	rpm
Max. continuous current (at 5000 rpm)	913	mA
Max. continuous torque (at 5000 rpm)	30	mNm
Max. permissible speed	15000	rpm
Peak torque (stall)	154	mNm
Mechanical parameters		
Rotor inertia	11	$kgm^2 * 10^{-7}$
Mechanical time constant	7	ms
Thermal resi. housing-ambient	8.6	C/W
Thermal resi. winding-housing	1.0	$°C/W$

scheme. Both schemes were evaluated considering three different set points. Fig. 5 shown FOC scheme and for FOC scheme and Fig. 6 the proposed sensorless FOC scheme.

The proposed FOC scheme presents less oscillations than FOC scheme with encoder. Also, the response of both are similar.

A comparison of the performance were realized employing the integral absolute error (IAE), the integral of squared error (ISE), the integral of time multiplied by absolute error (ITAE) and the integral of time multiplied by squared error (ITSE). The results for all these criteria are shown in Table 2.

Also, in Fig. 7 we have al these errors through time. In (a) IAE Criterion, (b) ISE Criterion, (c) ITAE Criterion and (d) ITSE Criterion are presented for FOC scheme and proposed sensorless FOC scheme.

All error criteria was evaluated considering a speed test reference of 30 rpm and 2 seconds of simulation.

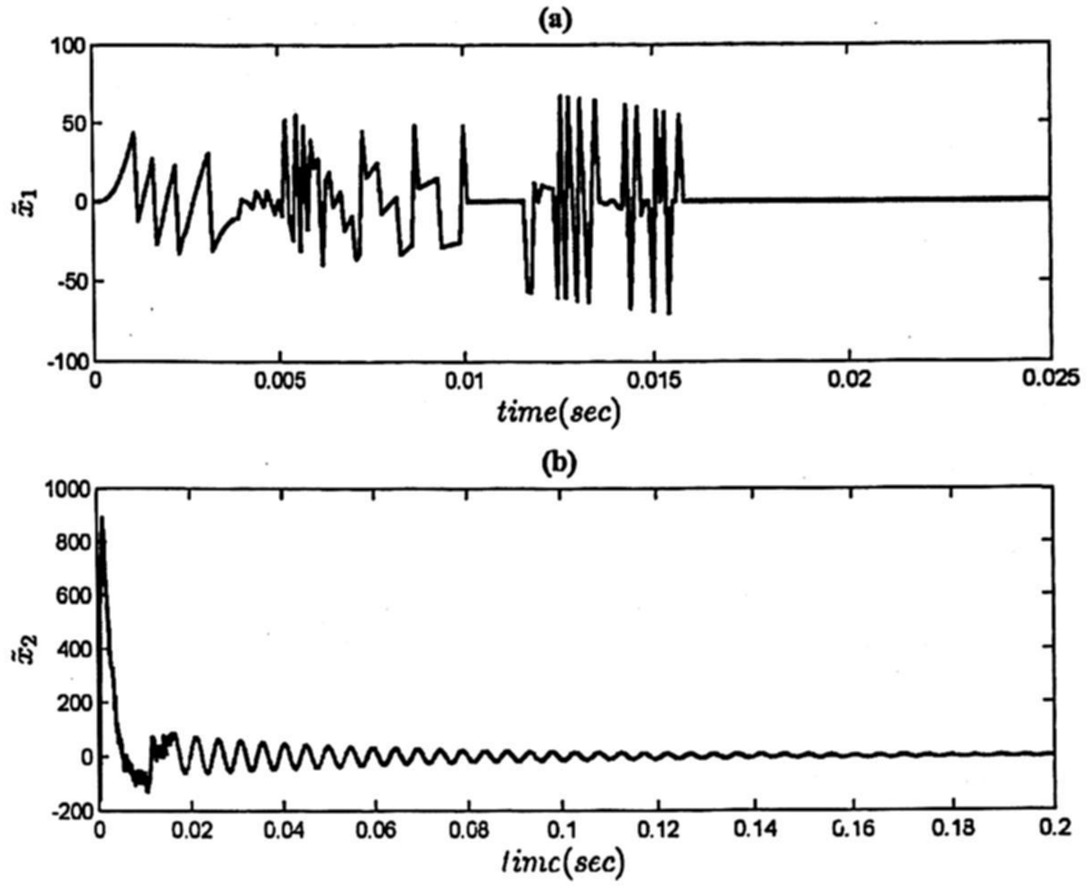

Fig. 4. Observer convergence: (a) Error estimation of $\hat{x}_1$ and (b) Error estimation of $\hat{x}_2$

Table 2. Error analysis.

	Error Criterion			
Parameter	IAE	ISE	ITAE	ITSE
Control with observer	0.091697	1.487097	0.00054	0.002007
Control without observer	0.60555	2.59605	0.509871	0.512472

5 Discussions and conclusions

This paper presents the design and simulation of a model-based speed sensorless for a PMSM. The proposed approach uses stator currents from the PMSM drive to estimate the rotor speed. A comparison of the response of the proposed model-based speed sensorless FOC scheme and a classic FOC scheme have been performed. Numerical simulations show that the proposed sensorless control presents less oscillations than FOC scheme, these oscillations appear due to sensor are sensitive to temperature, in addition, a bad estimation increase the error. The response of both systems are similar. In addition, the well performance is also demonstrated calculating the error criterion IAE, ISE, ITAE and ITSE. All the criterion of proposed FOC scheme are smaller than FOC scheme with encoder.

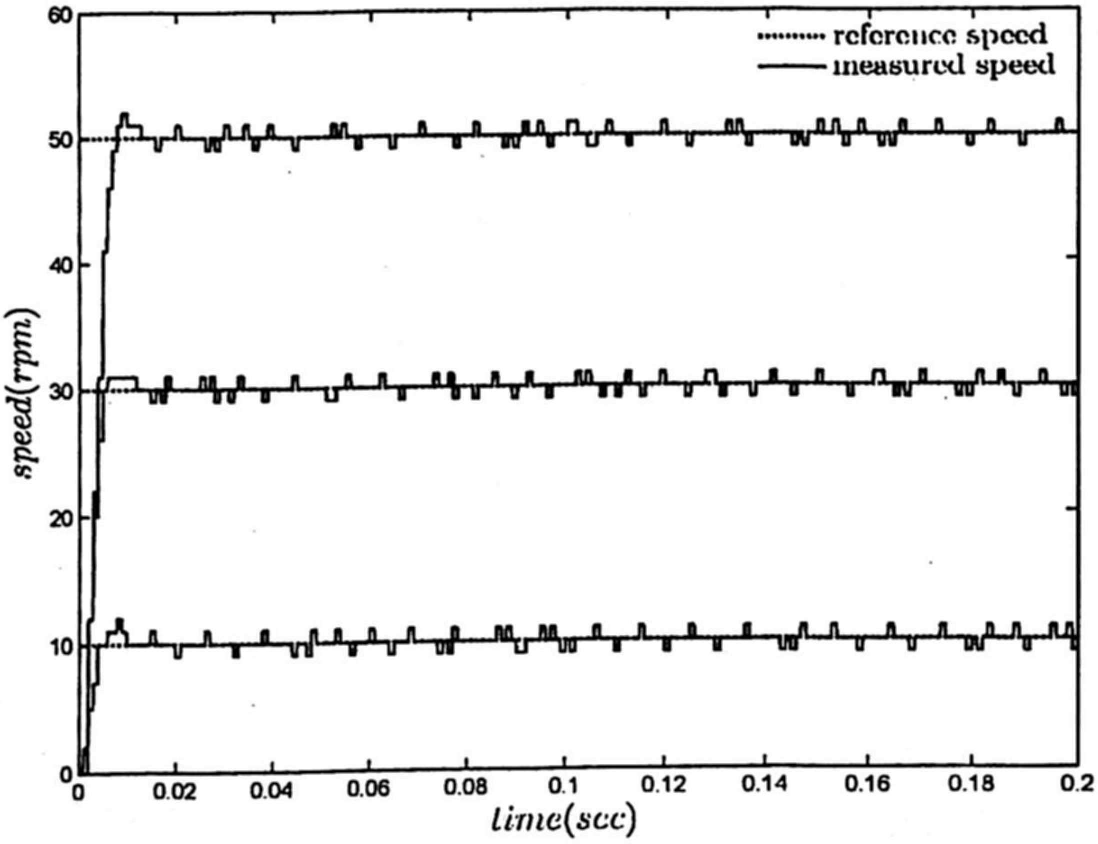

Fig. 5. Speed response of FOC.

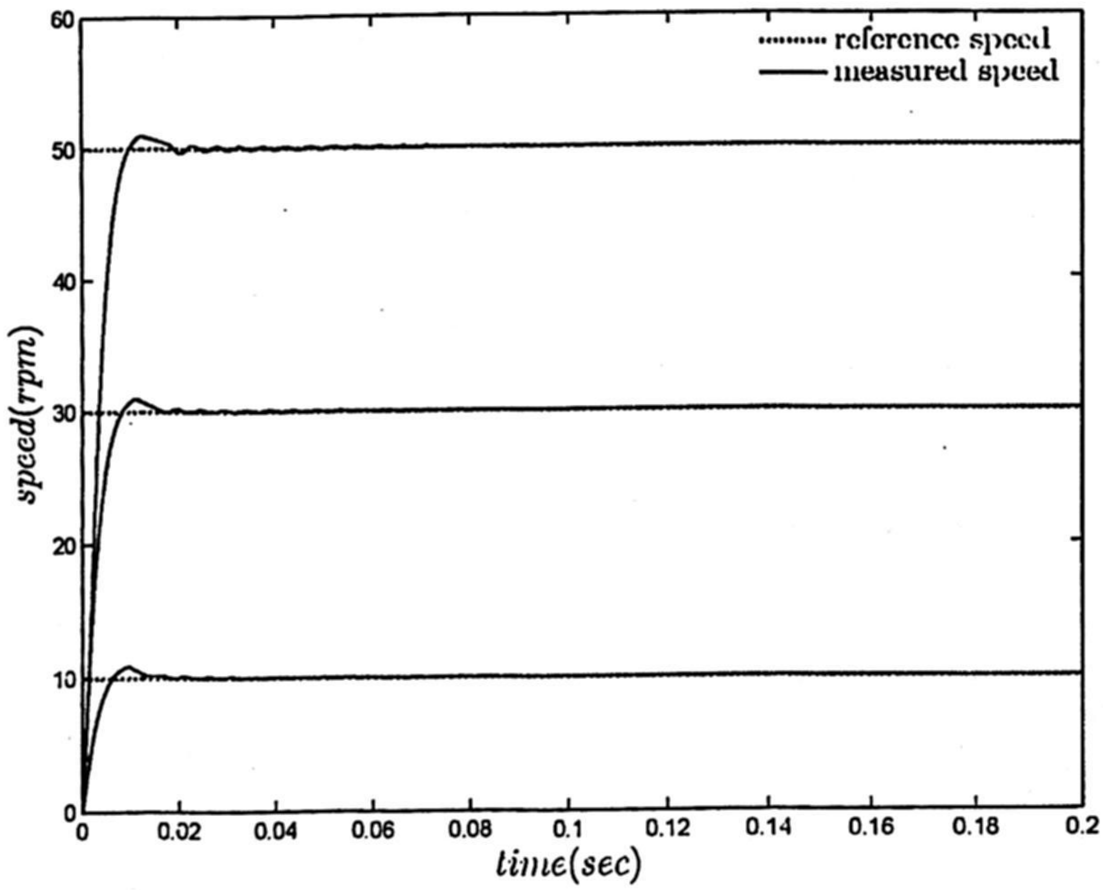

Fig. 6. Speed response of proposed sensorless FOC scheme.

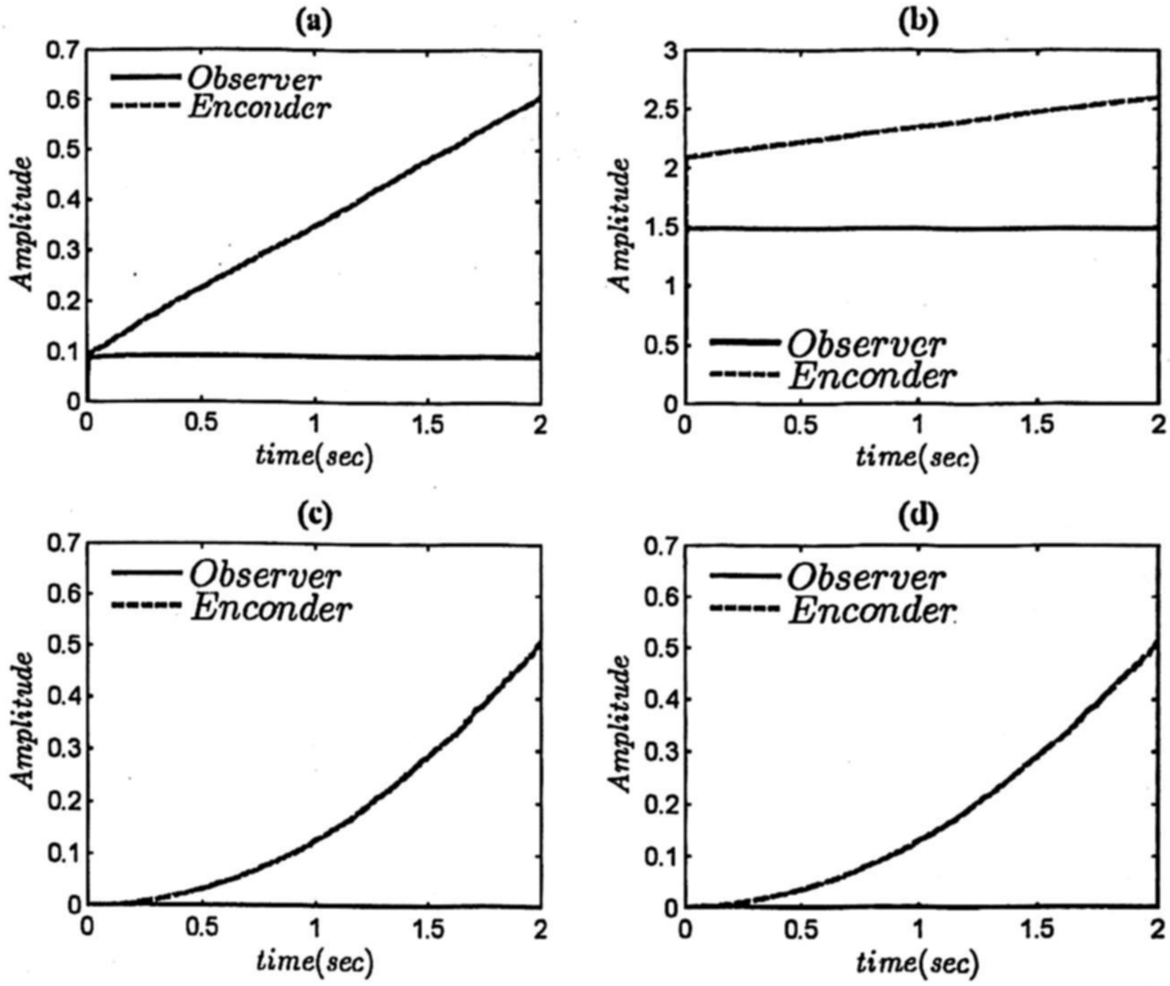

Fig. 7. Error analysis for both FOC schemes: (a) IAE Criterion, (b) ISE Criterion, (c) ITAE Criterion and (d) ITSE Criterion.

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